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Inter-Landau-level relaxation in two-dimensional electron gases at high magnetic fields

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Abstract. Inter-Landau-level relaxation in two-dimensional electron and hole gases in high magnetic fields is strongly affected by the extent, λ_z, of the electronic wavefunction perpendicular to the two-dimensional channel. As the field is increased, the suppression of the one-acoustic-phonon (cyclotron phonon) emission process that occurs when the phonon wavenumber at the cyclotron frequency exceeds λ_z^{-1} causes two-acoustic-phonon emission to replace one-phonon emission as the dominant relaxation process. The field at which this occurs has been calculated. The net reduction in relaxation rate is believed to be partly responsible for the sensitivity of the optical detection of cyclotron resonance in two-dimensional electron gas systems. In zero magnetic field, the two-phonon process should also contribute significantly to the energy and momentum relaxation of hot electrons.

1. Introduction

In this report we consider the effect of the two-phonon process on the inter-Landau-level acoustic phonon emission relaxation of magnetically quantized two-dimensional (2D) electron and hole gas systems. The suppression of the one-acoustic-phonon emission rate that occurs when the phonon wavenumber exceeds the inverse extent, λ_z^{-1}, of the electronic wavefunction $\varphi_0(z)$ across the 2D electron gas was noted in previous treatments [1]. However, the analysis was not extended to include higher-order processes.

The purpose of the present work is to point out that because of this suppression, there is a field B^* above which two-phonon emission at $\sim \omega_c/2$ becomes more efficient than cyclotron phonon emission at ω_c. The constraint $q > \lambda_z^{-1}$ set by momentum conservation is avoided through the emission of two oppositely directed phonons of approximately equal frequency. Ultimately this process should in turn be superseded by optic phonon emission accompanied by acoustic phonon emission (unless $n\omega_c = \omega_{LO}$). The existence of the two-phonon process should be observable in the electron temperature and, more directly, the phonon spectrum from the electrically heated magnetically quantized 2D electron gas system (including the 'hot spots' that occur at the current entry and exit points) and should also be apparent in their electrical mobility at elevated temperatures. However, even though the two-phonon process is stronger than the one-phonon process in this field range it remains relatively weak. We suggest this weakness is significantly responsible for the sensitivity of optical detection of cyclotron resonance in a 2D electron gas which has been the subject of recent experimental investigation in GaAs–AlGaAs heterostructures [2].

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2. One-phonon emission

In the single-particle approximation, the inter-Landau-level relaxation rate between two neighbouring levels $n+1$ and n at zero temperature has the form [3]

$$\tau_1^{-1} = \frac{2\pi}{\hbar} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} \delta(\hbar\omega_c - \hbar s q) |M_{n+1,n}^{(1)} V(q)|^2 [1 - \nu_n] \tag{1}$$

where the amplitude of the electron–phonon interaction,

$$|V(q)|^2 = \hbar(q^2\Xi^2 + \beta^2)/2\mu_0 s q$$

accounts for the deformation potential and unscreened† piezoelectric coupling in the material, and the matrix element (which is simply the Fourier transform of the electron wavefunction) is

$$|M_{n+1,n}^{(1)}|^2 = \left(\frac{(q_\parallel\lambda_B)^2}{2(n+1)}\right) \{L_n^1[\tfrac{1}{2}(q_\parallel\lambda_B)^2]\}^2 \exp[-\tfrac{1}{2}(q_\parallel\lambda_B)^2] F_{00}(q_z)$$

and contains the form factor determined by the extent λ_z of the subband wavefunction $\varphi_0(z)$:

$$F_{00}(q_z) = \left| \int \mathrm{d}z\, \varphi_0^2(z) \mathrm{e}^{\mathrm{i}q_z z} \right|^2 . \tag{2}$$

In these equations, the notations follow those of [4]: Ξ and β are the deformation and piezoelectric coupling constants respectively, s is the sound velocity averaged over longitudinal and transverse acoustic modes (in GaAs $s \simeq 0.5 \times 10^4$ m s^{-1}), μ_0 is the mass per unit cell, ν_n is the filling factor of the nth Landau level, L_n^1 is the normalized generalized Laguerre polynomial and λ_B is the magnetic length.

We have already noted that momentum conservation transforms into approximate selection rules for the phonon wavevectors. From the interplay of the polynomial and exponential factors in equation (1) one can see that the phonon emission matrix element is exponentially suppressed at wavenumbers $q_\parallel > \sqrt{n}\lambda_B^{-1}$ which is of the order of the zero-field Fermi wavenumber k_F of the electron gas. At high frequencies $\omega = \omega_c \gg s k_F$, this selection rule constrains the phonons to be emitted in directions very close to the normal to the plane [1, 5]. So, in this range, the emission rate is determined by the size of the subband matrix element F_{00} which falls rapidly for wavenumbers $q > \lambda_z^{-1}$. By integrating in equation (2) by parts its asymptotic form can be shown to be

$$F_{00}(q_z) \simeq \begin{cases} 4|\varphi_0'(0)|^4/q_z^6 = 4B_{00}(q_z\lambda_z)^{-6} & q_z\lambda_z \gg 1 \\ 1 & q_z\lambda_z \to 0. \end{cases}$$

For a triangular well, $B_{00} = 0.25$ and the extent λ_z can be related to the intersubband separation Δ_{10} as $\lambda_z = \sqrt{0.88\hbar^2/m\Delta_{10}}$ where m is the effective electron mass.

† As will be shown, the phonons with short wavelength (about that of the confining potential width λ_z) are relevant for the inter-Landau-level transition in the regime under consideration. This allows us to avoid effects of the electron–phonon interaction screening by 2D electrons due to the fact that λ_z is usually shorter than the screening length which is equal (in 2D systems) to donor related Bohr's radius.

Since, usually, $k_F \ll \lambda_z^{-1}$, the fall in F_{00} occurs at phonon wavenumbers $q \gg k_F$ and a good approximation can be made by integrating τ_1^{-1} over $q_\parallel$ with $q_z \simeq q = \omega_c/s$ to obtain

$$\tau_1^{-1} = [(1-\nu_n)/2\pi\hbar](\lambda_B^{-2}/\mu_0 s)\{[(\omega_c \Xi/s)^2 + \beta^2]/\omega_c\} F_{00}(\omega_c/s)$$

showing that the one-phonon relaxation rate is strongly suppressed when the cyclotron frequency ω_c exceeds the frequency $s\lambda_z^{-1}$ of a phonon whose wavelength equals 2π times the effective width of the confining potential. At high magnetic fields the relaxation rate becomes

$$\tau_1^{-1} = B_{00}\tau_D^{-1}(1-\nu_n)(s^5/\omega_c^4\omega_D\lambda_z^5)\sqrt{\hbar/2m\lambda_z^2\omega_D}[1 + (\tau_D/\tau_P)(2ms^2\omega_D/\hbar\omega_c^2)] \tag{3}$$

where ω_D is the Debye frequency of the acoustic phonon branch, and τ_D and τ_P are the relaxation times, for deformation and piezoelectric coupling respectively, of an electron with energy around $\hbar\omega_D$. In GaAs τ_D and τ_P both have values of about 5–10 ps [4] and, since $2ms^2\omega_D \ll \hbar\omega_c^2$ in the region of interest, we can neglect the second term in the bracket showing that the deformation potential coupling dominates with

$$\tau_1^{-1} \simeq 0.5(1-\nu_n)(\omega_D/\omega_c)^4 \times 10^3\ \mathrm{s}^{-1}$$

(this is the estimation for heterostructures with the areal electron density $1 \times 10^{15}\ \mathrm{m}^{-2}$).

3. Two-phonon emission

The significance of two-phonon emission arises through the suppression of the one-phonon process that occurs at high magnetic fields. In the two-phonon process the energy transferred to the lattice is equally shared between phonons of oppositely directed wavevectors, so reducing the constraints imposed by momentum conservation. The electron from the initial state $|i\rangle$ emits the first phonon and makes a virtual transition to some excited state; it then relaxes to the final state $|f\rangle$ by emitting the second phonon. In high magnetic fields, the electronic momentum $p \simeq \hbar\omega_c/2s$ transferred to each phonon results in large intermediate state energies $\varepsilon_t \simeq p^2/2m \simeq (\hbar\omega_c)^2/8ms^2 \gg \hbar\omega_c$.

The emission rate for the second-order process is calculated in the usual way [6] with

$$\tau_2^{-1} = \frac{2\pi}{\hbar}\int \frac{\mathrm{d}\boldsymbol{q}\,\mathrm{d}\boldsymbol{q}'}{(2\pi)^6}|M^{(2)}_{n+1,n}V(q)V(q')|^2[1-\nu_n]\delta(\hbar\omega_c - \hbar sq - \hbar sq') \tag{4}$$

where the matrix element

$$M^{(2)}_{n+1,n} = \sum_{|t\rangle}\left[\frac{\langle i|\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}|t\rangle\langle t|\mathrm{e}^{\mathrm{i}\boldsymbol{q}'\cdot\boldsymbol{r}}|f\rangle}{\hbar(\omega_c - sq) - \varepsilon_t} + \frac{\langle i|\mathrm{e}^{\mathrm{i}\boldsymbol{q}'\cdot\boldsymbol{r}}|t\rangle\langle t|\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}|f\rangle}{\hbar(\omega_c - sq') - \varepsilon_t}\right]$$

includes all possible intermediate states $|t\rangle$. Since $\varepsilon_t \gg \hbar\omega_c$, we neglect other terms in the denominator of $M^{(2)}_{n+1,n}$ and obtain

$$|M^{(2)}_{n+1,n}|^2 \simeq [8ms^2/(\hbar\omega_c)^2]^2|\langle i|\mathrm{e}^{\mathrm{i}(\boldsymbol{q}+\boldsymbol{q}')\cdot\boldsymbol{r}}|f\rangle|^2.$$

This has a maximum at $Q = q + q' = 0$ and since it decreases rapidly as Q deviates from zero, we approximate it by a delta function giving

$$|M^{(2)}_{n+1,n}|^2 \simeq \frac{\gamma_{00}}{\lambda_z}\left(\frac{2\pi}{\lambda_B}\right)^2 \delta(Q) \qquad \gamma_{00} = \lambda_z \int \mathrm{d}z\, \varphi_0^4(z).$$

In a triangular well $\gamma_{00} = 0.42$.

The integration of equaton (4) is now straightforward and gives

$$\tau_2^{-1} = 8\gamma_{00}\tau_{\mathrm{D}}^{-1}(1-\nu_n)(\tau_{\mathrm{D}}^{-1}s\omega_{\mathrm{c}}/\omega_{\mathrm{D}}^3\lambda_z)[1 + 8(\omega_{\mathrm{D}}ms^2/\hbar\omega_{\mathrm{c}}^2)(\tau_{\mathrm{D}}/\tau_{\mathrm{p}})]^2 \tag{5}$$

and since the second term in the bracket is again small in the region of interest we obtain for a typical GaAs heterostructure

$$\tau_2^{-1} \sim 0.5(1-\nu_n)(\omega_{\mathrm{c}}/\omega_{\mathrm{D}})10^8\ \mathrm{s}^{-1}.$$

Of particular note is that this two-phonon relaxation rate increases with magnetic field while the one-phonon rate τ_1^{-1} strongly decreases and from the two expressions we can estimate the magnetic field B^* for which $\tau_2^{-1} > \tau_1^{-1}$. We find that for a sheet density $n_s \sim 1 \times 10^{15}$ m^{-2}, the two-phonon process becomes the dominant relaxation mechanism at $B^* \sim 3$ T and that B^* increases with n_s to around 15 T at 1×10^{16} m^{-2} because of the strong decrease in τ_1^{-1} with the confining potential width parameter.

Analogous behaviour to this can possibly occur in the phonon emission from a hot 2D electron gas system in zero magnetic field. At low electron temperatures T_{e}, the phonon emission mainly occurs at wavenumbers $\sim k_{\mathrm{B}}T_{\mathrm{e}}/\hbar s$ which are small compared with both k_{F} and λ_z^{-1}. However, as T_{e} is increased, one-phonon emission becomes initially constrained to directions close to the normal to the plane by the in-plane selection rule and then strongly suppressed when $k_{\mathrm{B}}T_{\mathrm{e}}/\hbar s$ exceeds λ_z^{-1}. Two-phonon emission can then become the significant process in energy relaxation before ultimately being superseded by optical phonon emission.

4. Intersubband relaxation and optical detection of cyclotron resonance

The nature of the acoustic phonon emission in 2D systems in high magnetic fields has a strong influence on the possibility of optical detection of cyclotron resonance. The reduction in the net relaxation rate at ω_{c} arising from the suppression of the one-phonon process increases the likelihood that an electron in an excited Landau level will relax to nearer energy levels, should there by any, rather than undergo an inter-Landau-level transition. This is illustrated by arrow I in figure 1. An electron excited to the ground subband $n = 3$ Landau level prefers to relax to the $n = 0$ Landau level of the higher subband rather than to the ground subband $n = 2$ Landau level. Since the sample is exposed to visible light, the electron in the $n = 0$ level can rapidly recombine with a photo-created hole at a rate much faster than the intersubband relaxation rate [7, 8] and recombination from the higher subband should be even faster than that from the ground subband since the higher subband has a more extended wavefunction and so a greater average overlap with a hole. So the parallel process in which the electron in the higher-subband $n = 0$ level recombines after making a transition to the ground subband $n = 2$ level should be relatively weak. The effect of all this is that at these fields, and because of the one-phonon suppression, cyclotron absorption should result in an

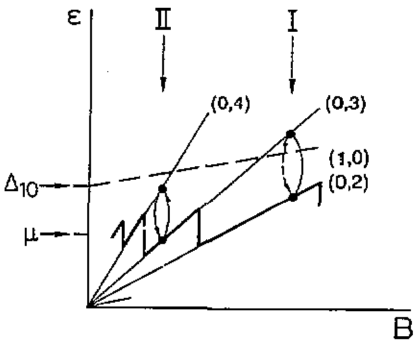

Figure 1. Fan plot which illustrates the possibility of optical detection of cyclotron resonance in 2D electron gases. We consider the case when the chemical potential μ in the system is smaller than the intersubband splitting Δ_{10}, so that there is no equilibrium occupation of the higher subband. Arrow I shows the regime where cyclotron resonance should be detectable through changes in luminescence and arrow II where it should not.

increase in the luminescence intensity from the excited subband together with a decrease in the intensity from the ground subband, so providing optical detection of cyclotron resonance. This technique is not expected to be possible at lower fields such as those indicated by arrow II since now the higher-subband level lies above any significantly populated Landau levels. These features have been demonstrated experimentally in GaAs–AlGaAs heterostructures containing a delta-doped layer of acceptors [2].

5. Summary

It has been shown that at high magnetic fields, the strong suppression of the one-phonon relaxation process between the Landau levels of a 2D electron gas (or 2D hole gas) results in its replacement by the two-phonon process as the dominant mechanism of energy relaxation. In a GaAs–AlGaAs heterostructure with sheet density 1×10^{15} m^{-2} this should occur for fields $B > B^* \sim 3$ T. For narrow-gap semiconductors the effect should occur at even lower fields because of the smaller effective masses and so larger cyclotron frequencies.

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